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ABSTRACT

Answers to the following questions were sought in this study: (1) Does the interview technique provide information that cannot be obtained from the usual normative approach?; (2) Does the interview technique provide information leading to the revision of mathematics test questions? Are the revised questions better than the original version in specified ways?; and (3) Does the interview technique provide information about the extent to which the student has been exposed in a mathematics course to the topic, concept, or skill that is central to the correct solution of a particular question? The College Board Mathematics Level II pretest was administered to 10 students in a senior mathematics course, to 30 juniors in a mathematics honors course, and to 35 juniors and seniors in mathematics courses. Fifteen selected students were then interviewed as to their methods of problem solution. The original pretest was then administered to the pretest population, and a pretest consisting of 16 questions identical to those in the original pretest and 9 questions that were revisions was also administered. Results of the study show that the answer to question (1) above was "yes"; in answer to question (2), nine questions in the pretest (items 2, 10, 14, 18, 20, 21, 23, and 24) were revised; and the answer to question (3) was a definite "yes." Appendixes provide the Item Interview Record and Test Interview Record, Procedures Used in Carrying Out the Study, Score Distributions for Total Group and Interview Group, Item Analysis for Total Group, Item Analysis for Interview and Pretest Groups and Interview Results for Each Item and Graphs. (DB)

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TEST DEVELOPMENT REPORT

TDR71-4 APRIL 1972

ED 067397

A COMPARISON OF INTERVIEW AND NORMATIVE ANALYSIS OF MATHEMATICS QUESTIONS

S. Irene Williams
Chancey O. Jones

Test Development Division, ETS

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A COMPARISON OF INTERVIEW AND NORMATIVE ANALYSIS OF MATHEMATICS QUESTIONS

by
S. Irene Williams
and
Chancey O. Jones

This study was sponsored by the College Entrance Examination Board and was conducted by Members of the Mathematics Department at the Test Development Division at Educational Testing Service.

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Miss Harriett L. Frankel who edited the final copy.

The school administrators, teachers, and students who so willingly participated in this study.

INTRODUCTION AND PURPOSE

The study on which this report is based originated with the request of the College Board Committee of Examiners in Mathematics that the usual statistical analysis of new test questions be supplemented by an attempt to find out what a student actually thinks as he solves the questions. The committee suggested that the conjectures about thought processes that are stimulated by item analysis results should be checked occasionally, if not regularly, against information obtained by the in-depth interviewing of students. At its April 1970 meeting, the committee recommended that a small-scale study be conducted to evaluate the merit of this approach. Arrangements were therefore made for members of Educational Testing Service to interview students in May 1970.

This study permitted the comparison of two theoretically different kinds of information about performance on test questions—the clinical and the normative. Standard item analysis provides information about the average performance of typical groups, but not about the procedure by which the examinee arrives at an answer or about the specific characteristics of a question that result in its being easy or difficult. In contrast to the normative information based on the statistical analysis of group data is the clinical information derived from the in-depth interviewing of individual students. The study reported here attempted to assess the usefulness of supplementing the normative information provided by item analysis with the clinical information derived from in-depth interviewing. The authors of this report found no studies that used the interview technique for this purpose, although they did not make an exhaustive search of the literature. Three studies (Bloom and Broder, 1950; Connolly and Wantman, 1964; and Gentile, 1966) made use of the "think-aloud" approach; however, none of these studies had as its purpose the revision of individual test questions.

The purposes of the study are summarized by the following questions:

1. Does the interview technique provide information that cannot be obtained from the usual normative approach?

2. Does the interview technique provide information leading to the revision of mathematics test questions?

Are the revised questions better than the original versions in any one of the following ways?

a. The revised version has appreciably better statistical characteristics than the original version when both are subjected to a standard normative item analysis.

b. The students, when interviewed, say that the revised version is clearer than the original version.

c. The interviewers result in the correction of an error in the question.

3. Does the interview technique provide information about the extent to which the student has been exposed in a mathematics course to the topic, concept, or skill that is central to the correct solution of a particular question?

METHOD

The College Board Mathematics Level II pretest used in this study consists of 25 questions to be administered in 40 minutes. This pretest is not typical of most pretests designed for the Level II population in that it contains more questions that test sophisticated concepts or terminology. Approximately one-third of the questions were specifically included by the committee to obtain information concerning the understanding of these concepts and the knowledge of technical terminology.

Two structured documents for recording information during the interviews were prepared: an Item Interview Record and a Test Interview Record. A copy of each of the forms is in Appendix A.

For the purpose of systematically organizing and recording student interview information, a notebook consisting of 25 Item Interview Records (one for each item in the pretest) and one Test Interview Record was prepared for each of the 15 interviews. In addition, each interview was tape recorded.

Two alternative procedures for collecting data were considered: (1) asking students to think aloud as they attempted to solve the problems in the pretest and (2) interviewing selected students after a group administration of the test under regular conditions. It was decided to follow the latter alternative because this more nearly replicated actual test-taking conditions. (See Appendix B for details of procedures used in carrying out the study.) Before the test was administered, the students were told the purpose

of the study without reference to the specific testing program. They were also told that if they were interviewed, they would be asked to recall their methods of solution and would be given their test book so that they could refer to the questions and any notes they had written in the book.

When it was pointed out that this study provided students with a chance for involvement in testing processes that affect them, they seemed to be motivated to perform well on the test. In fact most of the students seemed eager to be interviewed.

Table 1 describes the schools and the students involved in the study. More detailed statistical summaries may be found in Appendix C.

It can be seen from these data that the sample of schools ranged from quite strong to relatively weak. Within each school, the interview sample was chosen on the basis of test scores and information provided by the teacher. At each school an attempt was made to include in the interview sample at least one student who scored above the average student tested at that school, one who scored about average, and one who scored below average. It was soon discovered, however, that not much information about the methods of solution was obtainable from the students scoring below 10, so more able students were used as the study progressed. The distribution of scores in the interview sample ranged from 5 to 24. The distribution of scores appears in Appendix C. The interview sample included both boys and girls and minority-group representation.

After the interviews had been completed, the following two tests were administered to the usual pretest population for comparative purposes:

1. The original pretest that had been administered to the interviewees.

2. A pretest that consisted of 16 questions identical to questions in the original pretest and 9 questions that were revisions of questions in the original pretest.

The items from the two pretests, with the exception of four equating items that are not printed here because of the necessity for maintaining their security for possible future use, are included in Appendix D. Following each item are summaries of the statistical characteristics of the item for the interview group, for the group that took the original pretest, and for the group that took the revised pretest and a discussion of the results of the interview.

Table 1

Description of Schools and Students

School	Description of School	Description of Students	Students Tested			Students Interviewed		
			Number	Mean Score*	Score Range*	Number	Mean Score*	Score Range*
I	Public Suburban	Select seniors in a senior mathematics course	10	10.5	6-16	4	12.5	8-16
II	Public Suburban	Juniors in a mathematics honors course	19	11.2	5-16	3	14.3	13-16
III	Private Suburban	Juniors in a mathematics honors course	11	15.3	9-24	4	18.2	14-24
IV	Public Urban	Juniors and seniors in mathematics courses	35	6.8	2-15	4	9.8	5-15
Total			75	9.4	2-24	15	13.7	5-24

* Number of correct responses

SUMMARY AND CONCLUSIONS

The results of this study can be summarized in part by reconsidering the questions that were posed earlier in this report.

Question 1. Does the interview technique provide information that cannot be obtained from the usual normative approach?

From the evidence gathered in the 15 interviews, the answer to this question is "yes." For each of the questions, the interviewers gained some insight into the thought processes of the students as they solved the problems. In addition, the following general statements can be made as a result of interviewees' comments regarding individual questions.

a. Qualifiers or restrictive phrases such as $xy \neq 0$ that are necessary to solve the problem were in general ignored by the interviewees. As a rule they realized that the qualifiers were included in the wording of the questions for a purpose but did not always understand the purpose. They usually attempted to select the "best answer" in some sense completely disregarding the qualifiers. Although these qualifiers are ignored, it may still be necessary to retain them in some of the questions in order to make the questions mathematically sound.

b. Students at this level do not in general understand sophisticated mathematical phraseology such as "all but finitely many" and "if and only if." Because of the lack of mathematical sophistication of students at this level, perhaps such terminology should be avoided whenever possible.

c. Frequently students did not solve a problem using the method the item writer intended, especially if the intended method was an "insightful" one. Generally they attempted to apply a routine technique or procedure to a given problem rather than to think of a quick or insightful way of solving it. Unless questions can be devised that can only be solved by nonroutine methods, so-called insightfulness will probably not be a contributing factor in the scores of most students.

d. There is a tendency on the part of some students to substitute options whenever possible; however, many students resorted to this method only if they were unable to think of a direct approach to the problem. It is the opinion of the interviewers that the method of substitution is not

necessarily inappropriate since it indicates that the student has some understanding of the concepts involved; however, the specifications for a test probably should indicate the maximum allowable number of questions that can be solved by substitution.

Question 2. Does the interview technique provide information leading to the revision of mathematics test questions?

As a result of the interviews, 9 questions in the pretest were revised (items 2, 10, 14, 18, 20, 21, 22, 23, and 24). The original and revised versions of the items can be found in Appendix D. Only 4 of these revisions (items 10, 14, 18, and 23) resulted in substantial improvements in the statistical characteristics of the item for the pretest population. The lack of conclusive "statistical improvement," however, may just be an indication that the normative procedure is not as sensitive to difficulties that students have with nuances of wording as is the interview technique.

Question 3. Does the interview technique provide information about the extent to which the student has been exposed in a mathematics course to the topic, concept, or skill that is central to the correction solution of a particular problem?

The answer to this question is a definite "yes," although it raises the statistical question of generalizability to the candidate population from a limited sample. Certainly powerful clues are obtained through the interview process which might be corroborated by means of a candidate survey, or to some extent by an item analysis.

In addition, the following general statements can be made regarding the interviewees' reaction to the test as a whole.

1. The interviewees in general seemed to think that the test was at an appropriate level of difficulty, although a few of the students who had extremely low scores on the test considered it very difficult.

2. Although most interviewees found some topics tested that were unfamiliar to them, they stated that the coverage of the test was for the most part appropriate for them.

3. Most of the interviewees stated that they had sufficient time; however, most of them also indicated that they had skipped questions that they were unsure of or that they thought were too time-consuming.

4. Most interviewees indicated that they thought the test was fair, although a few students considered one or two of

the questions "tricky" or unreasonable. None of the students were concerned about the fact that they were unable to cope with some of the questions. They seemed to accept the idea that "not all students are expected to answer all of the questions."

5. Students generally objected to questions with a format like that of question 18. They felt that they might have knowledge about the concepts involved in the question and be able to ascertain whether or not one or more of the statements is true or false and still not obtain the correct response to the question.

SUGGESTIONS FOR FUTURE RESEARCH

It is the opinion of the interviewers that the information obtained from this study could be obtained by interviewing fewer students provided the students are selected from a variety of schools. For this particular study, in which a major purpose was to obtain information about the cognitive processes used in solving mathematics questions, just as much could have been learned from interviewing one average and one above-average student from each school. Very little was learned from the poor students; however, this might not be the case if the interviewing were being done for a different program or for a different purpose.

It is also the opinion of the interviewers that the item interview and test interview forms (Appendix A) used in this study should be revised. The item interview form was too complex; it was impossible for one person to complete the entire form during the interview. If the study were to be redone, number 3 would probably be deleted from the form. An attempt would also be made to structure the interview more by devising specific questions about each of the items to ask all students being interviewed. Many of the questions on the test interview form asked for information that duplicated that already obtained in the item-by-item interview. This form would probably be reduced to the questions contained in Section II.

Because of the amount of ETS staff time that would be required to conduct interviews on the large quantities of items that are pretested each year, it would not be practical to use the interview procedure for all items. The interviewers are of the opinion, however, that conducting this kind of study periodically is justified because of the kinds of information that can be obtained about the questions and the mathematics curricula in the schools. The interviewers also gained insight into the methods of solution used by the students and the kinds of misconceptions they have. This kind of information is extremely helpful in writing test questions with attractive distractors. In addition, the tapes provide a means for other ETS staff members to obtain knowledge of students' thought processes in problem solving.

APPENDIX A

Item Interview Record

I. What major concepts and/or abilities are being tested?

(A) Solution of a simple first degree equation in one unknown involving a fraction.

1. Were the major concepts and/or abilities tested?

Yes ☐ No ☐

2. Was a different concept and/or ability tested? Yes ☐ No ☐

II. Technique of Solution

(A) Option selected A ☐ B ☐ C ☐ D ☐ E ☐ Right ☐ Wrong ☐ Omit ☐

(B) What method did you use to answer this question?

1. Notes on method used

2. Procedures followed

(a) stem → options ☐
(b) options → marked choice ☐

3. Reasons for students' choice

(a) eliminated some options — guessed ☐
(b) random guess ☐
(c) faulty reasoning ☐
(d) specific determiner ☐
(e) lack of necessary knowledge ☐
(f) misconception ☐
(g) computational error ☐
(h) ambiguity in the question ☐
(i) too time-consuming ☐
(j) correct answer not recognized because of form ☐
(k) could not find his solution among the options ☐
(l) mistake in marking answer sheet ☐
(m) other — specify ☐

Test Interview Record

I. Opinions about the items

A. Were the content and abilities tested appropriate to you? Yes ☐ No ☐
List of questions:

B. Do you think all of the questions were clearly stated? Yes ☐ No ☐
List of questions:

C. Do you think any of the questions were tricky? Yes ☐ No ☐
List of questions:

D. Do you think any of the questions took too long to work? Yes ☐ No ☐
List of questions:

E. Were there any questions you particularly liked? Yes ☐ No ☐
List of questions:

F. Were there any questions you particularly disliked? Yes ☐ No ☐
List of questions:

II. Opinions about the test

A. Do you think the test was too easy ☐ , too difficult ☐ , about right ☐ ?

B. Was enough time allowed to do the test? Yes ☐ No ☐

C. Other comments:

APPENDIX B

Procedures Used In Carrying Out The Study

Cooperation in conducting the study was solicited from four high schools within a 15-mile radius of Princeton: two suburban high schools, one private preparatory school, and an urban high school. Following are lines of communication used in the four schools:

1. Principal to Mathematics Coordinator
2. Curriculum Director to Mathematics Coordinator
3. Headmaster to Mathematics Department Chairman to Teacher
4. Principal to Vice-Principal to Mathematics Department Chairman.

The actual work in each school was done in close collaboration with the last person indicated in each of the four cases above.

Since the first person contacted in each school needed to describe the study either to the school board or to the mathematics staff members, a brief description of the purpose and nature of the study was prepared and sent to the schools. A copy of the description can be found at the end of this appendix.

A team of three ETS Mathematics Department members conducted the test administration and interviews in each school. This number proved to be ideal since several different functions had to be performed. First, the test administration was facilitated by having several people available to give directions, distribute and collect supplies, count test books, and proctor. Second, the availability of several persons expedited the rapid completion of tasks required to select the interviewees and arrange for the interviews. This selection was done in collaboration with the mathematics teacher. While the answer sheets were being scored by one ETS staff member, a second arranged the interview room with appropriately placed chairs and recording equipment and attached school and student identification to the cassettes. Still another staff member helped to set up the interview schedule for the rest of the day after the interviewees were selected. This schedule was made to fit the student schedules. A system of notifying and paging students also had to be established, since the length of individual interviews varied somewhat. Finally, during the interviews one person did the actual questioning and introductory remarks on procedure, another recorded information in the notebook, and still a third ran the recorder and assisted in the

questioning. Although a team of three seemed to be ideal for this study as it was structured, it might be possible to restructure the interview techniques in such a way that two people could adequately complete a similar study.

Generally, the schedule of activities during a day at a school was as follows:

1. Enter school before classes begin and proceed to the main office. Explain purpose of visit again.
2. Go to the mathematics department office.
3. Review procedures again with the mathematics teacher.
4. Go to classroom.
5. Explain purpose of study to students.
6. Administer test.
7. Collect materials.
8. Score answer sheets.
9. Select three or four interviewees with advice of teacher; set up interview schedule.
10. Ask permission of each interviewee to record interview.
11. Interview each student for 1 to 1-1/2 hours.

Memorandum for: SCHOOLS PARTICIPATING
IN A SMALL-SCALE ETS
RESEARCH STUDY

Subject: Brief description of an in-depth study of mathematics test questions

Date: May 7, 1970

From: Sheldon S. Myers
Chancey O. Jones

Up-to-now new test questions have been operationally pretested on random samples of appropriate groups. This pretesting then permitted the computation of normative information about the question in terms of per cent selecting the correct choice, the numbers selecting the other four choices, and measures of discrimination. This kind of information, while very useful for revising questions and developing final test forms, dealt entirely with group performance and did not tell us anything about what the questions measured, whether ambiguities existed, and what thought processes were stimulated by the questions.

This study is an attempt to assess the feasibility of supplementing the above approach with an in-depth investigation of what happens when individual students take a mathematics test. This approach is essentially clinical, in contrast to the normative approach of item analysis.

The steps in this study are as follows:

1. Administration (by Myers and Jones) in the morning of a 40-minute pretest of 25 questions to a class of 10-20 students who have had 3½ years or more of college preparatory mathematics.

2. Scoring of the answer sheets by Myers and Jones in the next half-hour.

3. Interview in the rest of the day 4 to 5 students from this class. The interviews will last, on the average, 60 to 90 minutes and will have the following purposes:

    a. Determine what method was used by the student in arriving at his correct or incorrect choices.

    b. Determine if the student misunderstood any part of a question.

c. Determine if he was unduly delayed in any question and the reason.

d. Determine if the student felt that any question was unfair.

4. If the student agrees, the interview sessions will be taped. The identity of students and schools will be protected.

5. The teacher will help us in advance to select the students to be interviewed. The interviews will be held at times of greatest convenience to the student's schedule.

Besides the purely professional objectives of the study in evaluating a technique and in gathering more information about questions, the study is also consistent with the current trend of greater student involvement in processes that affect them.

SSM:klb

APPENDIX C

Score Distributions for Total Group and Interviewed Group

SCORE*	FREQUENCY	
	TOTAL GROUP	INTERVIEW GROUP
24	1	1
18	1	1
17	1	1
16	5	2
15	2	1
14	7	3
13	4	1
12	2	2
11	5	
10	5	
9	6	
8	10	1
7	7	1
6	8	
5	6	1
4	2	
3	1	
2	2	
MEAN	9.4	13.7
TOTAL NUMBER	75	15
NUMBER FROM:		
SCHOOL 1	10	4
SCHOOL 2	19	3
SCHOOL 3	11	4
SCHOOL 4	35	4

*Number of correct responses

Item Analysis for Total Group

Key	No.	Omit	A	B	C	D	E	P+
C	1	0	0	5	70	0	0	93.3
E	2	6	1	3	16	2	47	62.7
A	3	2	63	4	2	2	2	84.0
C	4	16	5	1	25	2	26	33.3
B	5	0	8	26	2	10	29	34.7
C	6	9	5	10	42	6	3	56.0
D	7	12	12	5	7	37	2	49.3
E	8	10	6	2	0	6	51	68.0
C	9	18	1	6	31	5	14	41.3
E	10	30	7	5	1	6	26	34.7
D	11	2	4	1	3	53	12	70.7
B	12	7	2	23	11	9	23	30.7
D	13	17	1	4	5	43	5	57.3
A	14	35	4	9	4	7	16	5.3
C	15	15	14	14	13	4	15	17.3
A	16	2	44	9	5	8	7	58.7
B	17	21	13	15	8	10	8	20.0
E	18	30	29	1	0	3	12	16.0
A	19	21	15	6	4	20	9	20.0
D	20	28	2	2	6	21	16	28.0
B	21	21	40	7	0	3	3	9.3
E	22	21	25	6	7	8	8	10.7
A	23	49	8	6	6	4	2	10.6
C	24	8	3	31	19	4	10	25.3
E	25	31	11	4	7	4	18	24.0

APPENDIX D

Item Analysis for Interview and Pretest Groups and Interview Results for Each Item

Following are detailed statistical summaries and a description of the interview results for each question*. In the discussions that follow, a question is considered to be easy if approximately 80 per cent or more of the pretest population answered the question correctly, of middle difficulty if approximately 60 per cent answered correctly, moderately difficult if approximately 30 per cent answered correctly, difficult if approximately 20 per cent answered correctly, and quite difficult if approximately 10 per cent or less answered correctly.

Since the ability level of the population that took the original pretest differed somewhat from that of the population that took the revised pretest despite the random assignment of pretests to students, the observed statistical characteristics of the original version of an item and its revision cannot be directly compared. A comparison of the two groups with respect to four item statistics (numbers of students omitting the items, difficulty levels of the items, item-test correlations, and ability levels of students selecting the correct answer) is shown in the graphs in Appendix E. Because of the lack of an obvious pattern for unrevised items in item-test correlation** (r-biserial) and ability level plots, these statistics will not be considered in the discussions concerning the results for revised items. There does, however, seem to be a definite pattern in the number of omits and difficulty level plots. In the following discussion, those revised items that are clearly outside of the pattern for unrevised items in at least one of these plots will be considered to have substantially different statistical characteristics in its original and revised forms.

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*Four of the questions that appeared in the pretest were from extant forms of the College Board Mathematics Level II Achievement Test and were used to relate new data to existing data. These four questions are not included in this report.

**It should be noted that the criteria used in computing the r-biserials for the two pretests were somewhat different because of the revisions that had been made in some of the items.

Question 1. If $\frac{x}{0.2} = 5$, then x =

(A) $\frac{1}{25}$ (B) $\frac{1}{10}$ (C) 1 (D) 10 (E) 25

Interview Group

BASE N	OMIT	A	B	C	D	E	M_{TOTAL}	Δ_E SCALE	Δ_E	CRITERION
				*15						
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
1								100.0		

Pretest Group I+

BASE N	OMIT	A	B	C	D	E	M_{TOTAL}	Δ_E SCALE	Δ_E	CRITERION
235	0	4	10	197*	7	17	13.0	BOARD	12.3	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
1	0.0	11.5	10.0	13.3	10.6	12.6	1.00	0.84	9.0	0.26

Pretest Group II‡

BASE N	OMIT	A	B	C	D	E	M_{TOTAL}	Δ_E SCALE	Δ_E	CRITERION
235	0	3	24	188*	8	12	13.0	BOARD	12.8	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
1	0.0	12.0	11.0	13.6	9.3	10.5	1.00	0.80	9.6	0.43

This question was included in the test in order to determine the student's ability to solve a simple linear equation involving fractions. It appeared as the first item because it had been judged to be easy. All of the interviewees answered the question correctly; twelve of the interviewees multiplied both members of the equation by 0.2, (whereas the remaining three replaced 0.2) with $\frac{1}{5}$, obtaining $5x = 5$, and then divided both members by 5. Some of the interviewees were surprised to find such an easy question on the test but did not object to it.

The data obtained from regular pretesting indicated that the question was easy since a majority of the students answered the question correctly. The topic is evidently quite familiar to the pretest population.

+Pretest Group I is the population that took the original version of the test.

‡Pretest Group II is the population that took the revised version of the test.

Question 2. If the following instructions to a computer are carried out in the order specified, what value of S should be written in instruction 6 ?

1. Let S = 0.
2. Let X = 5.
3. Let the new value of S equal the old value of S plus the value of X.
4. Increase X by 2.
5. If X < 8, go back to instruction 3. Otherwise, go on to instruction 6.
6. Write the final value of S.

(A) 0 (B) 5 (C) 7 (D) 9 (E) 12

Interview Group

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	1		1	2		*11				
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
2								73.3		

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	15	1	29	55	34	101*	13.0	BOARD	15.7	IS025
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
2	12.5	11.0	11.6	12.0	11.3	14.6	1.00	0.43	13.7	0.45

Question 2. If the following instructions to a computer are carried out in the order specified, what value of S should be written in instruction 6 ?

1. Let S = 0.
2. Let X = 5.
3. Let the new value of S equal the previous value of S plus the value of X.
4. Increase X by 2.
5. If X < 8, go back to instruction 3. Otherwise, go on to instruction 6.
6. Write the final value of S.

(A) 0 (B) 5 (C) 7 (D) 9 (E) 12

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	9	6	31	55	33	101*	13.0	BOARD	15.7	IS025
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
2	12.3	12.7	12.0	11.5	12.9	14.3	1.00	0.43	13.7	0.34

For some time the committee has been interested in finding out the extent to which candidates have been exposed to computer instruction in order to determine the appropriateness of including computer questions on the examination. This question was deliberately included in the pretest to obtain that kind of information. Of the fifteen students interviewed, seven students had been exposed to some computer instruction and solved the problem on that basis with five of them obtaining the correct answer. Of the eight who had had no exposure to the computer, one student omitted the question and seven treated it as a task in logical reasoning with six of these solving the problem correctly. It is obvious that although a computer background might be helpful it is not necessary for solving this problem. Consequently the results of this question did not give the committee all of the information it had hoped to obtain.

Since eleven of the fifteen interviewees selected the correct answer, a question of this kind that can be answered with or without knowledge of the computer seems to be an appropriate question to include on the test.

Several interviewees were confused by the use of the word "old." They were not sure whether this referred to the value of S that was initially stated or to the value that they had just obtained. As a result of their comments, the word "old" was replaced by the word "previous," and both versions of the question were pretested on the usual pretest population.

The results of the pretesting indicated no appreciable difference in the statistical characteristics of these two versions. However, the infrequency with which the item was omitted by the regular pretest population supports the surmise based on the interviews that the question is an appropriate one for the intended population.

Question 4. $\frac{\cos 50^\circ}{\sin 40^\circ} =$

(A) -1 (B) 0 (C) 1 (D) $\tan 10^\circ$ (E) $\cot 10^\circ$

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Interview Group					*11		4				
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	4								73.3		

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group I	235	38	6	5	76*	35	75	13.0	BOARD	16.5	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	4	12.6	13.0	12.8	15.1	10.6	12.3	1.00	0.32	14.9	0.47

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group II	235	33	13	5	70*	39	75	13.0	BOARD	16.7	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	4	12.9	10.5	11.2	15.4	11.8	12.0	1.00	0.30	15.1	0.52

The committee deliberately included this question in order to see whether students would recognize the equality of cofunctions of complementary angles. Of the fifteen interviewees, eleven answered the question correctly. Of these, four answered on the basis of cofunctions of complementary angles, five used the identity $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ without thinking about complementary angles, one used $\frac{\cos 50^\circ}{\sin 40^\circ} = \frac{\cos (45^\circ + 5^\circ)}{\sin (45^\circ - 5^\circ)}$ and used the sum formulas, and one sketched a right triangle and used the definition of trigonometric functions. The four who answered the question incorrectly thought that $\frac{\cos 50^\circ}{\sin 40^\circ}$ is equal to $\cot (50^\circ - 40^\circ)$.

For the regular pretest population the question was moderately difficult. "cot 10°," the most popular distracter was selected by about the same number of students as answered the question correctly.

Question 5. The set of all real x such that $\sqrt{x^2} = -x$ consists of

(A) zero only
(B) nonpositive real numbers only
(C) positive real numbers only
(D) all real numbers
(E) no real numbers

Interview Group	BASE N	OMIT	A	B	C	D	E	M_{TOTAL}	Δ_E SCALE	Δ_E	CRITERION
			3	*8		2	2				
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
	5								53.3		
Pretest Group I	BASE N	OMIT	A	B	C	D	E	M_{TOTAL}	Δ_E SCALE	Δ_E	CRITERION
	235	1	32	64*	9	36	93	13.0	BOARD	16.9	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
	5	16.0	13.9	15.1	12.8	12.4	11.5	1.00	0.27	15.5	0.43
Pretest Group II	BASE N	OMIT	A	B	C	D	E	M_{TOTAL}	Δ_E SCALE	Δ_E	CRITERION
	235	5	27	68*	5	44	86	13.0	BOARD	17.7	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
	5	9.0	13.3	14.9	12.6	12.7	11.8	1.00	0.29	15.2	0.40

The committee included this question on the test to determine whether students would associate $\sqrt{x^2}$ with $|x|$ and whether they were familiar with the convention that $\sqrt{\ }$ denotes the non-negative root. Among the interviewees, eight answered correctly; three of these used the fact that $\sqrt{\ }$ denotes the non-negative square root, two used the concept of absolute value, two selected the correct answer by examination of the available options, and one substituted numbers such as -1, 0, 1, and chose the correct answer.

Two of the seven interviewees who answered the question incorrectly thought that -x was a negative number and since $\sqrt{\ }$ is positive chose "no real numbers," two thought $\sqrt{\ }$ could be either positive or negative and selected "all real numbers," and the remaining three selected "zero" for different reasons: one of the latter had a very definite misconception that $|x| = -x$ only if $x = 0$; one believed $\sqrt{x^2}$ being positive and -x being negative implied $x = 0$; one substituted some numbers.

This was a moderately difficult question for the regular pretest population and the pattern of responses was somewhat similar to that of the interview group but with "no real numbers" being selected by more candidates than the correct answer was.

Although this question is probably appropriate, it appears that it involves two concepts about which many students are confused.

Question 6. For all x and y such that $xy \neq 0$, let $f(x, y) = \frac{xy}{x^2 + y^2}$.
Then $f(x, -x) =$

(A) $-x^2$ (B) $-\frac{1}{x^2}$ (C) $-\frac{1}{2}$ (D) 0 (E) $\frac{1}{2}$

Interview Group

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	1		1	*12		1				
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
6								80.0		

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	17	13	48	137*	13	7	13.0	BOARD	14.6	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
6	12.3	8.8	11.7	14.3	10.2	11.3	1.00	0.58	12.2	0.49

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	31	13	50	120*	14	7	13.0	BOARD	15.1	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
6	11.3	9.8	11.5	14.8	10.0	12.6	1.0[illegible]	0.51	12.9	0.56

The committee included this question to determine whether students could solve a simple substitution problem involving a function of two variables. Among the interviewees, twelve answered the question correctly and none of them were confused by the function notation; all twelve substituted -x for y and simplified. Two other interviewees also using this method made errors and one student was confused by the notation and omitted the question. In talking with the students, it appeared that little or no attention was paid to the qualifier "$xy \neq 0$." In general the interviewees did not consider this question to be difficult.

On the regular pretest population this question was of about middle difficulty. The relatively small number of omits seems to support the indication obtained from the interviewees that the notation was not confusing.

Question 8. If $x^4 - 1 = 80$, then $x^3 + x^2 + x + 1$ could equal

(A) -81 (B) -80 (C) -60 (D) -40 (E) -20

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Interview Group		1				1	*13				
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	8								86.7		
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	235	13	11	9	8	45	149*	13.0	BOARD	14.2	ISO25
Pretest Group I	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	8	12.0	9.8	10.6	10.1	10.4	14.4	1.00	0.63	11.7	0.59
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	235	23	6	9	4	58	135*	13.0	BOARD	14.7	ISO25
Pretest Group II	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	8	12.0	12.2	9.8	9.8	11.2	14.3	1.00	0.57	12.3	0.47

This question was included by the committee in order to determine whether the students would use $x^4-1 = (x-1)(x^3+x^2+x+1)$ in determining a value for the cubic expression. None of the interviewees used this factorization of the quartic in solving the problem. All of the interview group who attempted the question solved the equation $x^4=81$ and substituted. Of the thirteen who answered the question correctly, five immediately substituted -3 since all options were negative, one substituted -3 since x^3 was the dominating term, and seven others substituted 3 first and upon obtaining a positive value substituted -3. The one student who answered the question incorrectly substituted 3 and obtained 40 and assumed that substituting -3 would yield -40. One interviewee omitted the question. The question was not difficult for the interview group although they did not use the intended method.

The question was about middle difficulty for the pretest population and the number of omits was small, indicating that the topic was familiar to the pretest group; however, -40 was the most attractive distracter.

Question 10. A polynomial P(x) with real coefficients has three zeros. If two of the zeros are 0 and i, then P(x) could be

(A) $x^2 + 1$ (B) $x^3 - 1$ (C) $x^3 + 1$

(D) $x^3 - x$ (E) $x^3 + x$

Interview Group

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	5					*9				
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
10								60.0		

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	52	26	44	27	33	53*	13.0	BOARD	17.3	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
10	13.0	11.0	11.8	11.3	12.6	16.1	1.00	0.23	16.0	0.58

Question 10. A polynomial equation, P(x) = 0, with real coefficients has three roots. If two of the roots are 0 and i, then P(x) could be

(A) $x^2 + 1$ (B) $x^3 - 1$ (C) $x^3 + 1$

(D) $x^3 - x$ (E) $x^3 + x$

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	41	18	25	10	62	79*	1 .0	BOARD	16.3	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
10	12.2	10.6	10.6	11.3	11.8	15.9	1.00	0.34	14.6	0.66

The committee included this question to determine the familiarity of the terminology "zeros of a polynomial." The question was answered correctly by nine of the interviewees; six of these were familiar with the terminology and three were not familiar with it but correctly interpreted it to mean the roots of a polynomial equation. The remaining six did not understand the question; five of them omitted it and one guessed incorrectly. Six of the interviewees understood the concept of conjugate pairs of complex roots of polynomial equations with real coefficients and used this concept to solve the problem with two of them using the factor theorem and the other four eliminating options by substituting roots. The remaining three who answered the question correctly did so by substituting the given zeros in the options.

Since nine of the interviewees were not familiar with the terminology, it appeared that the question might not be appropriate for the intended population. The item was revised by changing "zeros of a polynomial P(x)" to "roots of a polynomial equation P(x) = 0."

On the regular pretest population the difficulty and the number of omits decreased substantially from the original version to the revised version. Although the revised version of the item was moderately difficult, it seem to be more understandable to the population than was the original form.

Question 11. $\sin^2 2\theta + \cos^2 2\theta =$

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1 (E) 2

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Interview Group						*15					
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	11								100.0		
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group I	235	29	8	6	6	145*	41	13.0	BOARD	14.3	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	11	11.3	9.9	10.3	10.8	14.2	11.4	1.00	0.62	11.8	0.47
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group II	235	40	11	9	17	119*	39	13.0	BOARD	15.1	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	11	12.0	10.7	10.0	11.4	14.4	11.9	1.00	0.51	12.9	0.44

The purpose of this question was to test the specific trigonometric identity, $\sin^2 x + \cos^2 x = 1$. All of the interview group answered the question correctly. Most of the students were not confused by the coefficient of θ although one student thought that the intent of 2θ was to confuse students and one thought it was "tricky" in that θ was not defined.

For the pretest population the item was of about middle difficulty. The most popular distracter was 2 which could be obtained by incorrectly assuming that $\sin^2 2\theta + \cos^2 2\theta = 2(\sin^2\theta + \cos^2\theta)$. A relatively small percentage of the students in the pretest population omitted this question.

Question 12. If n is a positive integer, then n! is divisible by 9 if and only if

(A) $n \geq 3$ (B) $n \geq 6$ (C) $n \geq 9$

(D) n is a multiple of 3 (E) n is a multiple of 9

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Interview Group				*8	1	2	3				
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	12								53.3		
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	235	12	3	43*	33	22	122	13.0	BOARD	17.8	ISO25
Pretest Group I	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	12	12.8	9.7	16.0	13.0	12.5	12.1	1.00	0.18	16.7	0.51
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	235	12	8	44*	25	30	116	13.0	BOARD	17.7	ISO25
Pretest Group II	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	12	12.1	9.1	16.5	13.4	13.1	11.9	1.00	0.19	16.5	0.61

The committee included this question in order to test the meaning of n! and the terminology "if and only if." The question was answered correctly by eight of the interviewees, incorrectly by six, and omitted by one. All but one of the students knew the meaning of n! but only one of the interviewees thought about "if and only if" in ruling out options. Most of the others who answered the question correctly felt that they selected the answer that was "best" in some sense which they could not always express or was most "complete" or "inclusive." The best student in the interview group answered the problem correctly without thinking about "if and only if."

It was a difficult question for the regular pretest population; however, very few students omitted the question. Almost three times as many students selected distracter (E) as selected the correct answer. Choice (E) would have been a correct answer for "if" instead of "if and only if." The data seem to support the impression obtained from the interview group that "if and only if" is not generally understood by students at this level.

Question 13. For $x > 1$, the graph of $y = \log_x x$ intersects the line $x = \pi$ at $y =$

(A) $-\pi$ (B) 0 (C) $\frac{1}{\pi}$ (D) 1 (E) π

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Interview Group		1	1		2	*9	2				
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	13								60.0		
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group I	235	48	6	14	37	102*	28	13.0	BOARD	15.7	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	13	12.3	10.7	11.4	11.6	14.6	11.5	1.00	0.43	13.7	0.44
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group II	235	71	7	22	20	96*	19	13.0	BOARD	15.8	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	13	12.7	10.0	11.0	11.0	14.5	12.2	1.00	0.41	13.9	0.39

The objective of this question was to determine whether students recognize that $\log_x x = 1$ for $x>1$ and that $y = \log_x x$ intersects $x = \pi$ at $y = 1$. Among the interviewed students, nine answered the question correctly, five answered the question incorrectly, and one omitted the item. It appeared from the interviews that the students either knew how to cope with the problem or guessed. The question was of middle difficulty for the interview group.

The question was slightly above middle difficulty for the pretest population with about one-fourth of this group omitting the question.

Question 14. If the matrix $A = [1\ 0\ -1]$ and the matrix $B = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$, then the matrix AB =

(A) $[-2]$ (B) $[3]$ (C) $\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & -1 \\ 0 & 3 \\ -1 & 1 \end{bmatrix}$ (E) $\begin{bmatrix} -1 & 0 & -1 \\ 0 & -3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

Interview Group

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	8	*3		1		3				
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
14								20.0		

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	69	26*	22	26	51	41	13.0	BOARD	18.6	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
14	13.7	15.1	11.5	13.3	11.6	12.8	1.00	0.11	17.9	0.30

Question 14. If matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then matrix AB =

(A) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ (E) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	53	28*	15	11	49	79	13.0	BOARD	18.5	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
14	12.5	14.2	13.1	14.7	12.8	12.8	1.00	0.12	17.7	0.18

This question was included by the committee in order to determine whether or not the topic of matrices is appropriate for this population. Only two of the interviewees had studied matrices in their course work with one of these answering the question correctly. Reports had been written on the subject by two other students and both answered the question correctly. Of the remaining eleven who were not familiar with the topic, three guessed incorrectly and eight omitted the question. This question was clearly inappropriate for this group; however, since some of the students indicated that 2 by 2 matrices might be more familiar to students, a new question was written to test multiplying 2 by 2 matrices rather than the multiplication of a 1 by 3 matrix and a 3 by 1 matrix.

Both of the questions were quite difficult for the pretest population probably because many students had not been exposed to the topic. The decrease in the number of omits from the original item to the new one on the regular pretest population substantiated the indication from the interviewees that students might be more familiar with 2 by 2 matrices than with nonsquare matrices; however, both questions were extremely difficult and were about the same level of difficulty. Although only a small percentage of the students omitted the question, the pattern of responses seems to indicate that many students were not familiar with matrix multiplication but attempted to devise a reasonable definition of multiplication.

Question 16. Which of the following is the set of all real numbers x such that $|x| - 5 \leq |x - 5|$?

(A) The set of all real numbers
(B) {5}
(C) {−5}
(D) $\{x: x \leq 0\}$
(E) $\{x: x \geq 0\}$

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Interview Group			*8	2	2	1	2				
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
	16								53.3		
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	235	3	123*	25	6	32	46	13.0	BOARD	15.0	ISO25
Pretest Group I	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
	16	13.0	14.3	10.8	10.5	11.8	11.8	1.00	0.52	12.8	0.43
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	235	7	123*	23	8	38	36	13.0	BOARD	15.0	ISO25
Pretest Group II	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
	16	9.0	14.2	12.1	10.8	11.0	12.9	1.00	0.52	12.8	0.39

The purpose of this question was to determine whether the students could find the solution of an inequality involving absolute value. The question was answered correctly by eight of the interviewees and incorrectly by seven. Those answering the question correctly did so by substituting negative, positive, and zero values into the expression. All of the others attempting to find the solution also used substitution but did not consider positive, negative, and zero values. The question was slightly above middle difficulty for this group.

For the pretest population, the question was also slightly above middle difficulty, and there were very few omits.

Question 17. If two different numbers are chosen at random from the set {1, 2, . . . , 6}, what is the probability that their sum is greater than 6 ?

(A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{5}{9}$ (D) $\frac{1}{2}$ (E) $\frac{2}{5}$

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Interview Group		6	1	*6	1		1				
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	$P+$	Δ_O	r_{bis}
	17								40.0		
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	235	31	53	63*	31	39	16	13.0	BOARD	16.9	ISO25
Pretest Group I	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	$P+$	Δ_O	r_{bis}
	17	13.9	12.2	14.6	12.4	11.4	12.8	0.99	0.27	15.5	0.33
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	235	31	53	54*	20	52	25	13.0	BOARD	17.3	ISO25
Pretest Group II	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	$P+$	Δ_O	r_{bis}
	17	14.0	12.1	13.8	14.1	12.5	12.0	1.00	0.23	16.0	0.15

The objective of this question was to measure the students' understanding of simple probability. The question was answered correctly by six of the interviewees, but one had guessed and one had gotten the right answer for the wrong reason. The item was omitted by six, answered incorrectly by three. Six of the students stated that they did not think that they had sufficient background for solving this problem, and at least two other students misread the question. The question was above middle difficulty for the interview group.

The question also proved to be moderately difficult for the regular pretest population although the number of omits was not excessive.

Question 18. Let f and g be functions such that $g(f(x)) = x$ for all x. If (5, 0) is a point on the graph of $y = g(x)$, which of the following equations could define f ?

I. $f(x) = x - 5$
II. $f(x) = 3x + 5$
III. $f(x) = x^3 + 5$

(A) I only (B) II only (C) III only
(D) I and II only (E) II and III only

Interview Group

BASE N	OMIT	A	B	C	D	E *3	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
ITEM NO. 18	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+ 21.0	Δ_O	r_{bis}

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	51	114	15	14	14	24*	13.0	BOARD	18.8	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
18	12.6	13.2	11.1	11.6	12.6	15.2	0.99	0.20	18.1	0.31

Question 18. If f and g are inverse functions and if (5, 0) is a point on the graph of $y = g(x)$, which of the following could define f ?

I. $f(x) = x - 5$
II. $f(x) = 3x + 5$
III. $f(x) = x^3 + 5$

(A) I only (B) II only (C) III only
(D) I and II only (E) II and III only

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	56	84	8	9	11	66*	13.0	BOARD	16.9	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
18	12.9	11.6	10.0	10.9	13.4	15.5	1.00	0.28	15.4	0.51

This question was included to test the concept of inverse function by the definition $g(f(x)) = x$ without stating that f and g are inverse functions. Four of the interviewees recognized that f and g are inverse functions, but one of these did not realize that the point (5,0) on the graph of g implies that the point (0,5) is on the graph of f. Of the remaining eleven students who did not recognize that f and g were inverse functions, three attempted to answer the question but selected an incorrect answer, whereas the other eight omitted the question. Some of the interview group did not like this item type which consists essentially of three true-false questions, all of which must be answered correctly to receive credit.

It appears that the testing of inverse functions using definition instead of terminology was not appropriate for the interview group. However, the concept of inverse functions does appear to be appropriate for students at this level. Since many of the students indicated that they were familiar with the concept of inverse functions but had not recognized its formal definition, the item was revised as shown above.

Both of the questions were considerably above middle difficulty for the regular pretest population; however, the item type may have contributed to some extent to the difficulty. Although the number of omits was approximately the same on both versions, the difficulty level dropped substantially from the original to the revised version.

This decrease in difficulty tends to substantiate the indication given by the interviewees that the concept of inverse functions is more familiar to students than is the formal definition.

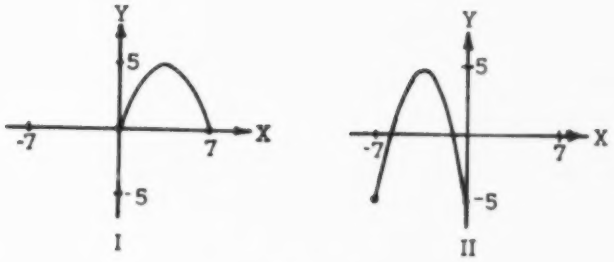

Figure 1

Question 19. In Figure 1, if the equation of the graph in I is $y = f(x)$, then the equation of the graph in II could be

(A) $y = 2f(x + 7) - 5$
(B) $y = 2f(x - 7) + 5$
(C) $y = 2f(x - 7) + 5$
(D) $y = 2f(x - 7) - 5$
(E) $y = 2f(-x) + 5$

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Interview Group		7	*6			2					
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	19								40.0		
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group I	235	25	42*	23	20	88	34	13.0	BOARD	17.8	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	19	12.4	15.0	13.7	13.8	12.0	12.6	0.99	0.18	16.7	0.34
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group II	235	39	35*	12	19	96	31	13.0	BOARD	18.2	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	19	13.1	15.3	13.3	12.5	12.7	11.9	0.99	0.15	17.2	0.36

The purpose of this question was to determine whether students could recognize the effect that translating a graph would have on the equation of the original graph. The question was answered correctly by six interviewees, incorrectly by two, and omitted by seven. The students answering correctly either used concepts of translation, substitution of points, or a combination of both. The item was somewhat above middle difficulty for this group. It seemed apparent in the interviews that some of the students were confused about the correspondence between the direction of shift involved in a translation and the sign of the number to be added to the abscissa or the ordinate.

The question was difficult for the pretest population. The fact that option D was selected by more students than was the correct answer tends to support the impression that a number of students are confused about the "sign" associated with a translation.

Question 20. Which of the following functions satisfy $|f(x)| \geq 1$ for all but finitely many numbers in the interval $0 < x \leq 2\pi$?

I. $f(x) = \sec x$
II. $f(x) = \csc x$
III. $f(x) = \tan x$

(A) I only (B) II only (C) III only
(D) I and II only (E) I, II, and III

Interview Group

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	1				*10	3				
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
20								66.7		

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	56	20	23	38	73*	21	13.0	BOARD	16.5	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
20	12.4	10.9	13.2	11.6	15.0	12.3	0.98	0.32	14.9	0.44

Question 20. For which of the following functions is $|f(x)| \geq 1$ for all x in the interval $0 < x \leq 2\pi$ for which f is defined?

I. $f(x) = \sec x$
II. $f(x) = \csc x$
III. $f(x) = \tan x$

(A) I only (B) II only (C) III only
(D) I and II only (E) I, II, and III

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	65	18	14	48	55*	31	13.0	BOARD	17.2	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
20	12.3	13.0	13.3	12.1	15.4	11.8	0.98	0.24	15.9	0.46

This question was included to test the range of trigonometric functions and the meaning of absolute value. The ranges of the given functions were considered by nine of the interviewees; six of these answered the question correctly and three incorrectly. Four considered the reciprocals of sec and csc and their ranges and drew conclusions about the ranges of sec and csc. All four of these students answered the question correctly; however, one of these incorrectly eliminated the tangent because it was not defined on the entire interval. The question was omitted by one and guessed incorrectly by one.

Almost without exception the students did not understand the phrase "all but finitely many" and tended to ignore it in solving the problem.

The topic was evidently a familiar one; however, because of the confusion regarding the wording the question was revised.

Both versions of the question were moderately difficult for the pretest population. However, again the item type may have contributed to some extent to the difficulty of the question. Revising the item did not appear to make any appreciable difference in the statistical characteristics of the item.

Question 21. $\lim_{t \to 0} \frac{2 - \sqrt{4 - t}}{t} =$

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 2 (E) 4

Interview Group

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	3	7	*4		1					
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
21								26.7		

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	22	150	27*	12	16	4	13.0	BOARD	18.5	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
21	12.3	12.9	14.8	14.1	11.4	11.8	0.98	0.12	17.7	0.26

Question 21. $\lim_{t \to 0} \frac{t}{2 - \sqrt{4 - t}} =$

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 2 (E) 4

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	46	143	6	9	9	18*	13.0	BOARD	19.2	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
21	12.9	13.0	10.0	10.9	13.2	15.8	0.98	0.08	18.7	0.37

This question was included in order to test the concept of limit and technique for determining a limit when both the numerator and denominator of a fraction tend to 0. Of the fifteen students interviewed only four used a correct technique; one correctly answered the question using L'Hospital's rule, and three rationalized the numerator with two of them obtaining the correct answer. Of the remaining eleven students, four substituted 0 in the numerator and, since it was 0, disregarded the denominator and therefore answered the question incorrectly; three others estimated the value of $\frac{2-\sqrt{4-t}}{t}$ for values of t close to 0 and guessed, with one guessing the correct answer. One additional student made a random incorrect guess and three students omitted the question. Most of the students seemed to have been exposed to the concept of limit but either did not remember or had not studied the technique required to solve this problem.

Since most students are familiar with rationalizing denominators rather than numerators the item was revised so that the limit could be obtained by rationalizing the denominator.

For the regular pretest population, both the number of omits and the difficulty level increased slightly from the original to the revised form. Both versions of this item were quite difficult. The concept of limit may be an appropriate topic for this group, but perhaps it should be tested with a less sophisticated technique.

Question 22. In a complex plane, the set of points z such that $|z| \leq 1$ is a region whose boundary is

(A) two parallel lines
(B) an equilateral triangle
(C) a square
(D) an ellipse
(E) a circle

Interview Group

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	4	2	1	3	1	4				
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	$P+$	Δ_O	r_{bis}
22								26.7		

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	38	79	11	37	24	41*	13.0	BOARD	17.8	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	$P+$	Δ_O	r_{bis}
22	13.1	13.2	11.6	12.8	11.6	14.1	0.98	0.18	16.7	0.18

Question 22. If a complex number, a + bi, is plotted in a coordinate plane as the point (a, b), then the set of points such that $|a + bi| \leq 1$ is a region whose boundary is

(A) two parallel lines
(B) an equilateral triangle
(C) a square
(D) an ellipse
(E) a circle

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	59	60	13	38	27	29*	13.0	BOARD	18.4	ISO25
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	$P+$	Δ_O	r_{bis}
22	13.5	11.4	12.5	13.4	13.1	15.1	0.96	0.13	17.5	0.32

The intent of this question was to determine the extent to which the complex plane and the absolute value of a complex number are understood by students at this level. Only two of the interviewees understood the concepts involved in this question. One of these replaced $|z|$ by $\sqrt{x^2+y^2}$, and the other interpreted z as an ordered pair and $|z|$ as distance. Most of the others trying the question guessed and four students omitted the question. It appears that these concepts were not appropriate for the interview group.

So few students understood what is meant by "complex plane" that the question was revised.

Both versions of this question were quite difficult for the pretest population and no appreciable differences were evident in the statistical characteristics. It appears from the pattern of responses that a number of students may have randomly guessed.

Question 23. Which of the following is the center of the conic section whose equation is $4x^2 - 9y^2 + 16x + 90y - 245 = 0$?

(A) (−2, 5) (B) (2, −5) (C) (2, 5)
(D) (5, −2) (E) (5, 2)

Interview Group

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
	8	*4	1	1	1	1				
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
23								26.7		

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	95	50*	30	25	21	7	13.0	BOARD	17.4	IS025
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
23	12.5	15.4	12.4	12.2	12.6	10.3	0.97	0.22	16.1	0.44

Question 23. Which of the following is the center of an ellipse whose equation is

$$4x^2 + 9y^2 + 16x - 90y - 245 = 0 ?$$

(A) (−2, 5) (B) (2, −5) (C) (2, 5)
(D) (5, −2) (E) (5, 2)

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
235	82	50*	34	29	12	9	13.1	BOARD	17.3	IS025
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P_+	Δ_O	r_{bis}
23	13.1	14.8	12.3	12.1	11.7	11.4	0.92	0.23	16.0	0.32

The intent of this question was to test the students' ability to find the center of a conic section. Five of the fifteen interviewees used the correct method of completing the square; four obtained the correct answer, and one selected the coordinates with the wrong signs. Of the remaining ten, two guessed incorrectly and eight omitted the question. Several of the interviewees were not familiar with finding the center of a conic section although they stated during the interview sessions that they had learned how to find the center of a circle in a coordinate plane.

Because of the interviewees' unfamiliarity with "conic section," this phrase was replaced with "ellipse." For the regular pretest population both versions were difficult. There was a noticeable decrease in the number of omits from the original to the revised version; however, there was no appreciable change in the level of difficulty.

Question 24. How many real numbers are solutions of the equation $x^2 = 2^x$?

(A) None (B) One (C) Three
(D) Four (E) Infinitely many

Interview Group

BASE N	OMIT	A	B	C	D	E	M_{TOTAL}	Δ_E SCALE	Δ_E	CRITERION
			8	*3		4				
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
24								20.0		

Pretest Group I

BASE N	OMIT	A	B	C	D	E	M_{TOTAL}	Δ_E SCALE	Δ_E	CRITERION
235	2	23	129	25*	11	37	13.0	BOARD	18.6	IS025
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
24	18.0	12.4	13.0	15.4	12.3	11.9	0.97	0.11	17.9	0.34

Question 24. Use the graphs of $y = x^2$ and $y = 2^x$ to determine the number of real solutions of the equation $x^2 = 2^x$.

(A) None (B) One (C) Three
(D) Four (E) Infinitely many

Pretest Group II

BASE N	OMIT	A	B	C	D	E	M_{TOTAL}	Δ_E SCALE	Δ_E	CRITERION
235	23	15	94	31*	10	36	13.0	BOARD	18.1	IS025
ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P_{TOTAL}	P_+	Δ_O	r_{bis}
24	14.4	11.9	13.2	14.0	11.3	11.4	0.89	0.15	17.1	0.16

The intent of this question was to determine the students' ability to find the number of solutions to an equation using a graphical method. Only three students in the interview group obtained the correct answer. Only one of these used a graphical method; the other two found two solutions by inspection, and one of these guessed there should be a negative solution and the other incorrectly thought 8 was a solution. The other students in the interview group either found one solution by inspection and selected "1" for the answer or found two solutions by inspection and guessed "infinitely many."

It seems apparent that students are not familiar with using graphs to determine the number of solutions of an equation of this type. In order to determine whether the students could find the number of real solutions if they were told to use graphs, the question was revised as shown above.

For the regular pretest population, there was an increase in the number of omits from the original to the revised version; however, there was no appreciable difference in the difficulty levels. Both versions of the item are quite difficult.

Question 25. Which of the following lines are asymptotes of the graph of $y = \frac{x-2}{x-3}$?

(A) x = 3 only
(B) y = 3 only
(C) x = 1 and y = 3
(D) x = 3 and y = 3
(E) x = 3 and y = 1

	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Interview Group							*7				
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	25								46.7		
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group I	235	0	67	21	22	13	61*	13.2	BOARD	16.6	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	25	0.0	12.4	11.5	11.7	10.5	15.8	0.78	0.33	15.0	0.58
	BASE N	OMIT	A	B	C	D	E	M TOTAL	Δ_E SCALE	Δ_E	CRITERION
Pretest Group II	235	0	69	18	21	9	48*	13.2	BOARD	16.9	ISO25
	ITEM NO.	M_O	M_A	M_B	M_C	M_D	M_E	P TOTAL	P+	Δ_O	r_{bis}
	25	0.0	13.5	11.1	12.0	11.7	14.6	0.70	0.29	15.5	0.28

The intent of this question was to determine whether students could find the vertical and horizontal asymptotes of the graph of an algebraic equation. Seven of the interviewees answered the question correctly, three answered incorrectly, and five omitted the item. One of the students answering the question correctly saw that the equation was undefined for x=3, solved for x in terms of y, and found that for y=1 the equation was also undefined. Two other students recognized the vertical asymptote and found the horizontal asymptote by finding the number that y approached as x became very large. The other four students also recognized the vertical asymptote, substituted values for x, and concluded that y got close to 1 but did not become 1. Three students found the vertical asymptote only and selected option A. The question was moderately difficult for the interview group.

The question was also moderately difficult for the pretest population. Option A was the most attractive distracter and was selected by more students than was the correct answer.

APPENDIX E

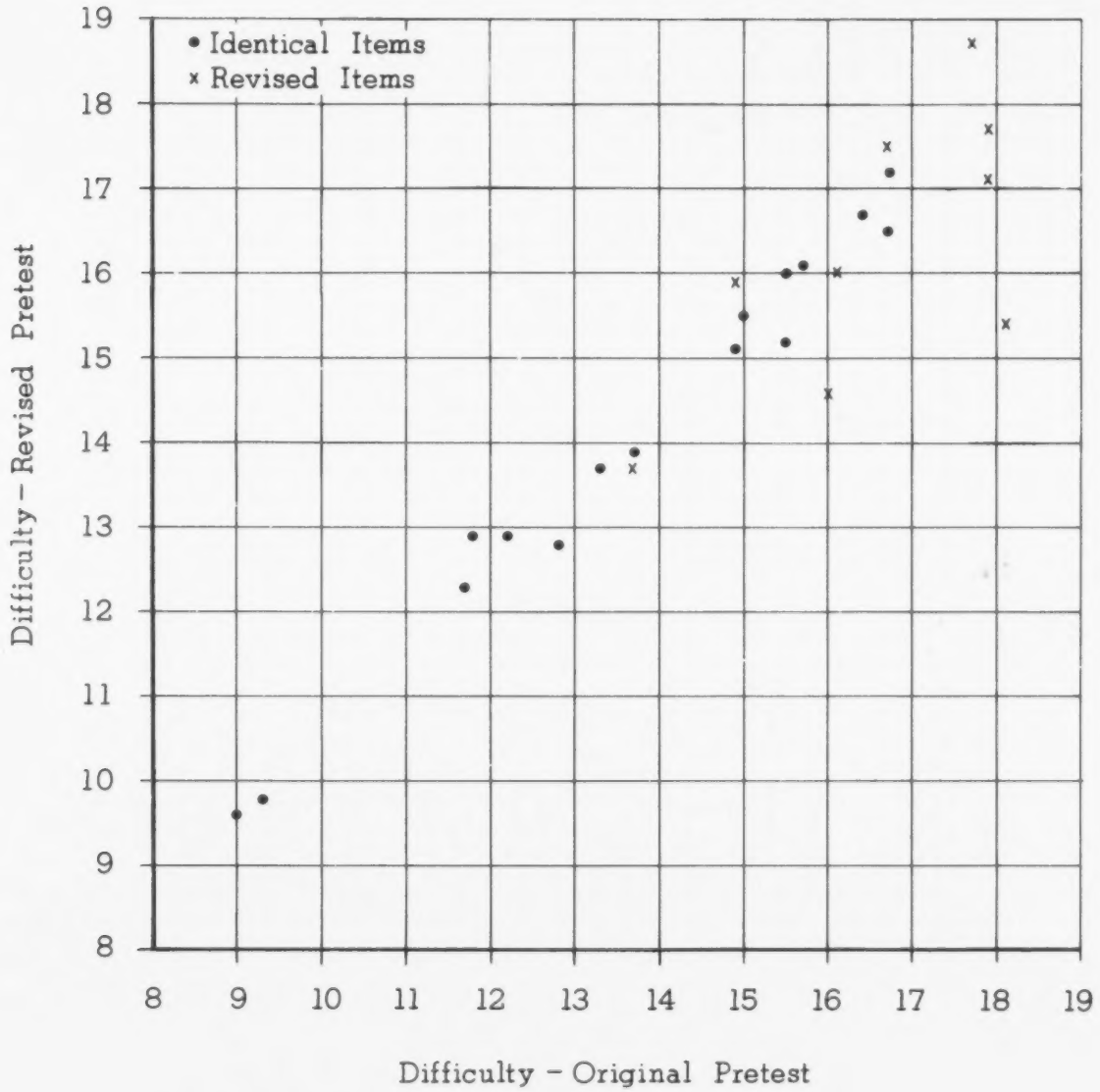

Fig. 1. Difficulties of Corresponding Items in Original Pretest and Revised Pretest

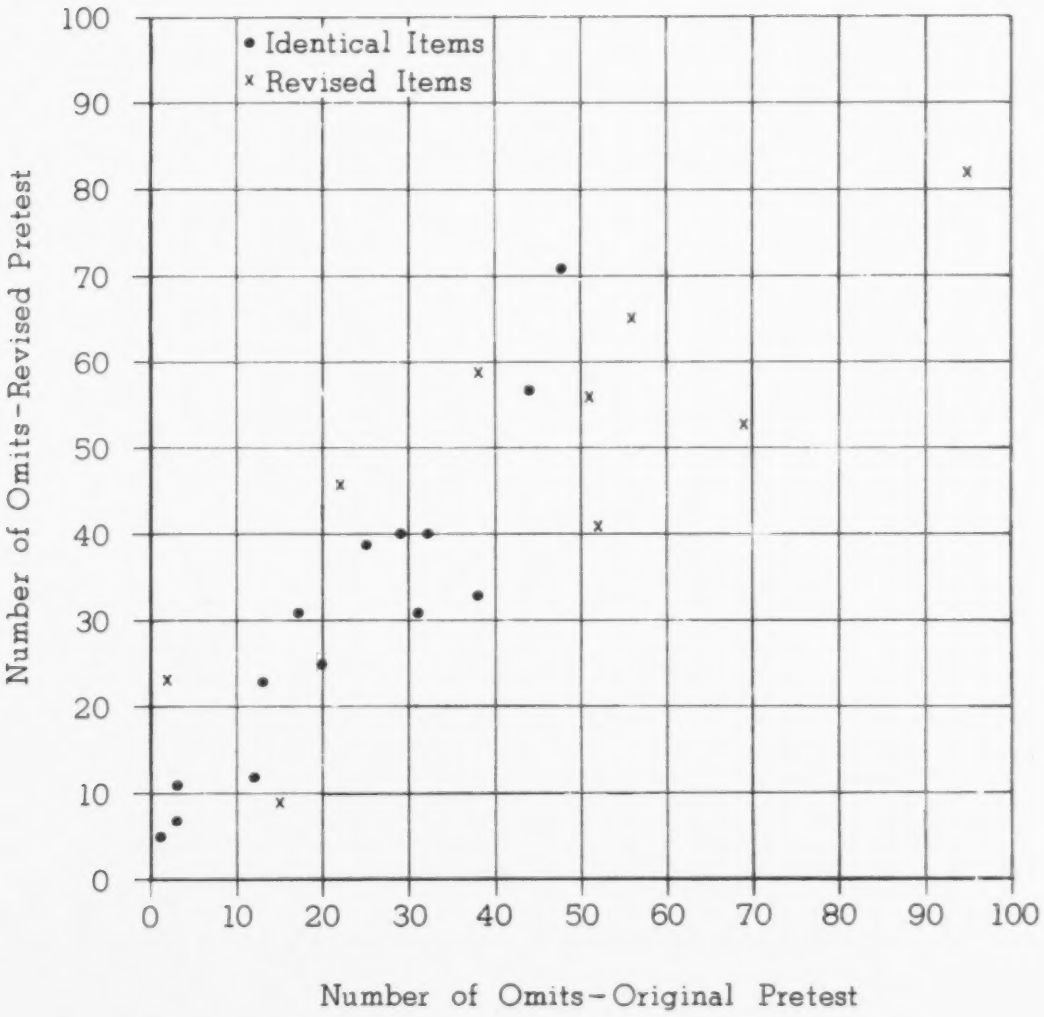

Fig. 2. Numbers of Students Omitting Corresponding Items in Original Pretest and Revised Pretest

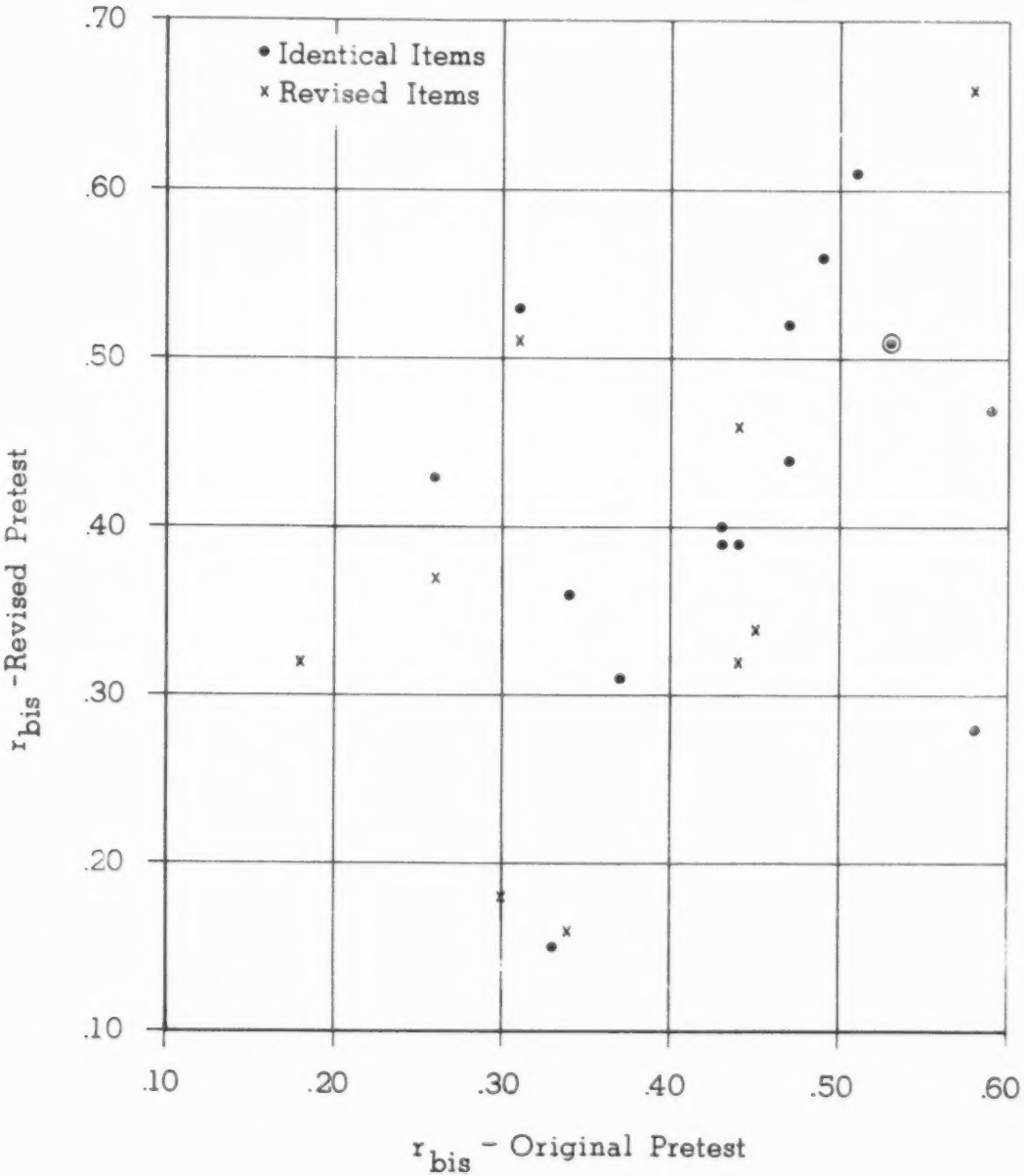

Fig. 3. Corresponding Item-Test Correlations in Original Pretest and Revised Pretest

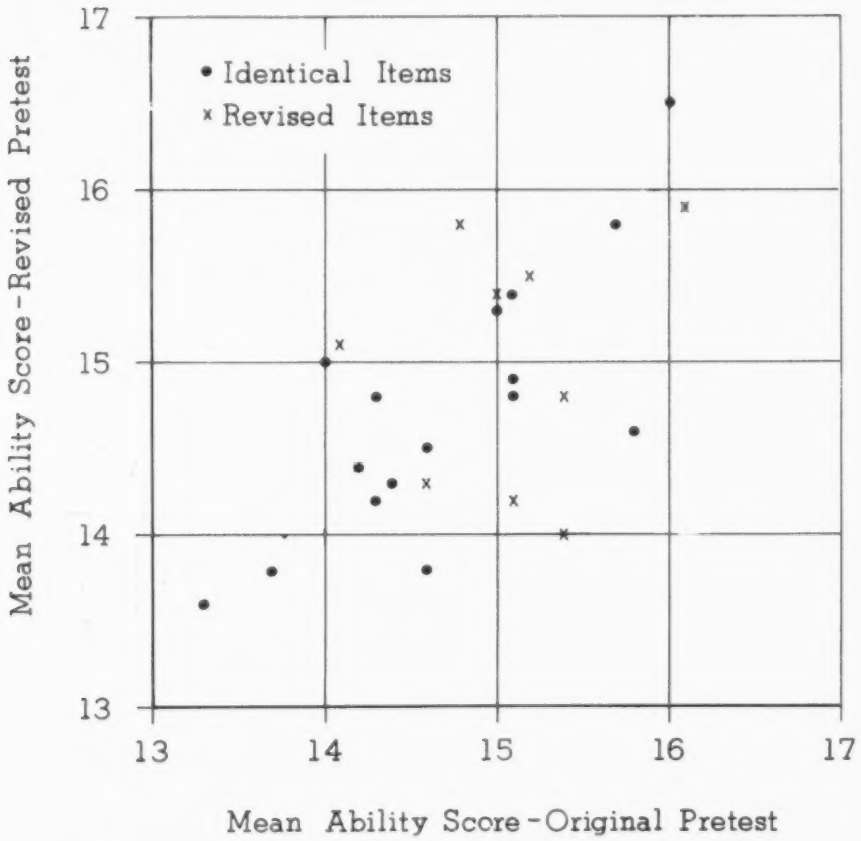

Fig. 4. Mean Ability Scores of Students Answering Corresponding Items Correctly on Original Pretest and Revised Pretest